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Note on Geometric Inferences from Algebraic Symmetry.

BY FRANK MORLEY.

If, on a function symmetrical in y, z, we perform an operation symmetrical in y, z, the result will be symmetrical in y, z.

From this simple principle we may infer geometric results not otherwise obvious. For instance, the equation of a quartic curve having 1 node and 2 cusps is

$$(yz + zx + xy)^2 = kx^2yz,$$

and from Plücker's equations it has 2 inflexions and 1 double tangent. The double tangent cannot pass through the node, and must therefore be, from the symmetry in y, z,

$$y + z = \alpha x.$$

The line joining the two inflexions cannot pass through the node, for the loop, being symmetrical in y, z, cannot have only one inflexion; hence this line must be symmetrical in y, z, or is

$$y + z = \beta x.$$

Again, one inflexional tangent is related to y just as the other is to z. Hence the line joining the points where they meet the curve again will be

$$y + z = \gamma x.$$

Also, from each inflexion one tangent can be drawn. The line joining the points of contact will be

$$y + z = \delta x.$$

These lines intersect on the line joining the cusps, $x = 0$.

The result, so far as concerns the double tangent and the line joining the inflexions, is stated in Salmon's Curves, §288, the proof being left to the reader. A special case is given as Exercise 143 in the Annals of Mathematics, June 1887. For another instance, take the sextic with 3 triple points, each the limit of a node and 2 cusps. Then Plücker's equations give, since $m = 6$, $\delta = 3$, $\varkappa = 6$,

$$\begin{aligned} n &= m = 6, \\ \iota &= \varkappa = 6, \\ \tau &= \delta = 3. \end{aligned}$$

The curve is

$$(yz + zx + xy)^3 + 9kx^2y^2z^2 = 0;$$

and since a double tangent cannot pass through a triple point, the 3 double tangents are

$$\begin{aligned} y + z &= ax, \\ z + x &= ay, \\ x + y &= az, \end{aligned}$$

i. e. a double tangent, the tangent at a triple point, and the line through the other triple points are concurrent. It is not easy to draw any certain inference about the 6 inflexions from the symmetry, but by forming the Hessian we can show that they lie on the conic

$$(3 - k)^2 u = 4k(x + y + z)^2,$$

writing

$$u \equiv yz + zx + xy;$$

and since all the intersections of the conic and sextic are inflexions, the conic must touch the sextic at the inflexions.

HAVERFORD COLLEGE, November 22, 1887.